DYNAMICAL (SUPER)SYMMETRY BREAKING

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Dynamical Symmetry Breaking (DSB) is a concept theorists rely on very often in the discussions of strong dynamics, model building, and hierarchy problems. In this talk, I will discuss why this is such a permeating concept among theorists and how they are used in understanding physics. I also briefly review recent progress in using dynamical symmetry breaking to construct models of supersymmetry breaking and fermion masses.

1 Hiearchy Problems, Stability of Hiearchy

There are many hierarchy problems in the Standard Model. A hierarchy is a puzzle if there is no apparent reason for one number to be much smaller than the other. To understand them is one of the main applications of the DSB. Here is the list of most important hierarchy problems:

1. Smallness of the electroweak scale: $v/M_{\rm Planck} \sim 10^{-17} \ll 1$.

2. Fermion mass hierarcy: $m_e/m_t \sim 2 \times 10^{-6} \ll 1$.

3. Flatness of the Universe. In order for the Universe now to be as flat as observed, it must have been extremely flat in the Early Universe at temperature T: $|\Omega - 1| \lesssim 3 \times 10^{-16} (1 \text{ MeV}/T)^2 \ll 1$.

4. Cosmological constant. Using the current "cosmic concordance" number, $\Lambda/M^4_{\rm Planck} \sim 10^{-123}$.

5. Ultimate hierarchy: H.M. $\ll$ Bill Gates.

None of the above hierarchies have obvious reasons for them, and our very existence crucial depend on them.

A very important question about a hiearchy is if it is stable. Certainly for the case of Bill Gates, it is; once there is an accumulated wealth, it reproduces itself and the hiearchy does not collapse easily (or even tends to grow). If there were any reasonable explanation behind a hierarchy, the hierarchy should be stable; otherwise it is either transient, accidental, or initial condition dependent and there is no hope to understand it. Only when the hiearchy is stable against small perturbations, we can get started to ask the question why.

Here is one examle of an unstable hierarchy: the electron mass in classical electromagnetism.[1] Since an electron generates a Coulomb field around it, and it feels its own Coulomb field, there is a self-energy for the electron

$$\Delta E \sim + \frac{e^2}{4\pi\epsilon_0 r_e}, \tag{1}$$

where r_e is the "size" of the electron. Thanks to Einstein, mass is nothing but the rest energy, and the observed mass is the sum of the "bare" mass and the rest energy

$$(m_e c^2)_{\rm obs} = (m_e c^2)_{\rm bare} + \frac{e^2}{4\pi\epsilon_0 r_e}. \tag{2}$$

The problem is that the mass is linearly divergent in the pointlike limit $r_e \to 0$. In fact, we know $r_e \lesssim 10^{-17}$ cm experimentally, and the above equation would look numerically like

$$0.511 = -9999.489 + 10000.000. \tag{3}$$

If the "bare" mass was different by 1%, the electron mass becomes 200 times larger! The smallness of the electron mass $m_e c^2 \ll e^2/(4\pi\epsilon_0 r_e)$ is the hierarchy problem in this case, and this is an unstable hierarchy; a small perturbation destroys the hierarchy. This implies that the classical electromagnetism is not applicable for distances below $e^2/(4\pi\epsilon_0 m_e c^2) \sim 10^{-13}$ cm.

It turned out that the quantum mechanics and the discovery of anti-particles made the hierarchy stable. The Coulomb self-energy discussed above can be depicted as a diagram where the electron emits a virtual photon (Coulomb field) and reabsorbs it (feels it). This gives a positive self-energy $e^2/(4\pi\epsilon_0 r_e)$. But now that we know the world is quantum mechanical and there exists the anti-particle of the electron, namely positron, we have to consider the following funny process. The vacuum constantly fluctuates to produce a pair of electron-positron together with a photon, where these particle survive within the time allowed by the uncertainty principle $\Delta t \sim \hbar/\Delta E = \hbar/(2m_e c^2)$ and annihilate back to the vacuum. When you put an electron in the vacuum, the electron sees the fluctuation, and sometimes it annihilates the positron in the vacuum fluctuation. Then the electron that was originally in the quantum fluctuation remains as a real electron. If you calculate the contribution of this process to the self-energy, you find $-e^2/(4\pi\epsilon_0 r_e)$. The leading linearly divergent pieces exactly cancel between these two contributions and the self-energy ends up with a sub-leading piece:

$$(m_e c^2)_{\rm obs} = (m_e c^2)_{\rm bare} + \frac{3\alpha}{4\pi}(m_e c^2)_{\rm bare} \log \frac{\hbar}{m_e c r_e}. \tag{4}$$

Now that the self-energy depends only logarithmically on r_e, the correction to the electron mass is only 9% even if the electron is as small as we can imagine: the Planck size $\sim 10^{-33}$ cm where the quantum field theory breaks down anyway. Because the self-energy is proportional the bare mass itself, a 1% perturbation in the bare mass results in a 1% change in the observed mass, and the hierarchy is stable. Once the hierarchy $(m_e c^2)_{\rm bare} \ll e^2/(4\pi\epsilon_0 r_e)$ is set, it stays including the self-energy contribution. The reason behind the stability is the chiral symmetry $\psi_e \to e^{i\theta\gamma_5}\psi_e$, which is exact in the massless electron limit, but is explicitly broken by the finite electron mass $m_e \neq 0$. This is why the self-energy, which violates chiral symmetry, comes out proportional to the violation of chiral symmetry in the theory, the electron mass itself. The hierarchy is made stable by doubling the number of particles.

We have learned that the smallness of the electron mass, or in general fermion mass hierarchy, is stable. Therefore we can hope to understand the hierarchy. Indeed one can write down models which naturally explain fermion mass hierarchy using approximate flavor symmetry as we will come back to later.

The problem which has been concerning many theorists is that the smallness of the electroweak scale is not a stable hierarchy. In the Standard Model, we have the Higgs potential

$$V = m_H^2 |H|^2 + \lambda |H|^4 \tag{5}$$

and the electroweak symmetry is broken spontaneously in the vacuum because the coefficient of the quadratic term is negative $m_H^2 < 0$. The electroweak scale is given by $v = \langle H \rangle = \sqrt{-m_H^2/2\lambda} = 174$ GeV. A self-energy diagram for the Higgs boson produces a virtual pair of top-quark which annihilates back into the Higgs boson. This diagram gives the correction

$$\Delta m_H^2 = -\frac{3h_t^2}{16\pi^2}\frac{1}{r_H^2}, \tag{6}$$

where $h_t \approx 1$ is the top quark Yukawa coupling and r_H the "size" of the Higgs boson. Because $\lambda \gtrsim 1$ violates perturbative unitarity, $|m_H^2| \lesssim (174 \text{ GeV})^2$ for the Standard Model to be well-defined. Then following the same logic as in the case of the electron mass, the Standard Model is not applicable to distance scales below $4\pi/\sqrt{3}v \sim 1$ TeV! If it is applied to much shorter distance scale, the smallness of the electroweak scale is spoiled by small perturbations.

The idea of supersymmetry is that the instability of the electroweak scale is solved by doubling the number of particles again.[1] In supersymmetry, we introduce superpartners to every particle in the Standard Model. The superpartner of the top quark $\tilde{t}$ contributes also to the self-energy of the Higgs boson. If you calculate it, you find

$$\Delta m_H^2 = +\frac{3h_t^2}{16\pi^2}\frac{1}{r_H^2}, \tag{7}$$

and the leading quadratically divergent pieces cancel. The total self-energy correction is then given by

$$\Delta m_H^2 = -6\frac{h_t^2}{16\pi^2}(m_{\tilde{t}}^2 - m_t^2)\log\frac{\hbar}{m_{\tilde{t}} c r_H}. \tag{8}$$

Hiearchy is now stable even for truly elementary (Planck-sized) Higgs boson, *as long as* $m_{\tilde{t}} \lesssim$ a few 100 GeV. This is why supersymmetry at sub-TeV scale is interesting; we can get *started* to ask why the electroweak scale is so small. In the Minimal Supersymmetric Standard Model, m_H^2 is positive in the supersymmetric limit, and the electroweak symmetry cannot be broken; only after supersymmetry is broken, m_H^2 can be negative. Therefore, breaking of supersymmetry induces the electroweak symmetry breaking. The question we would like to ask then is why the supersymmetry breaking scale is so low.

2 Dynamical Symmetry Breaking at Work

We have discussed that both the fermion mass hierarchy (thanks to chiral symmetry) and the smallness of the electroweak scale (if supplemented by supersymmetry) are stable, so that we are now entitled to ask why these hiearchies exist in nature. But neither chiral symmetry nor supersymmetry explains the origin of hierarchy. Both of them have to broken in nature; otherwise all quarks and leptons are massless, or the electroweak symmetry is not broken. They have to broken by a very small amount. This is where the dynamical symmetry breaking comes in.

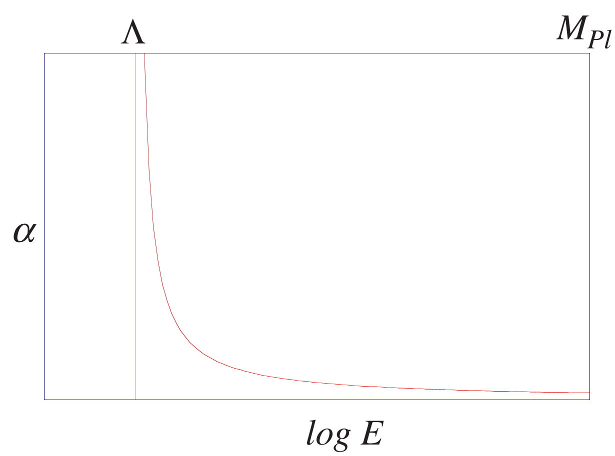

Figure 1: A schematic plot of the running coupling constant in an asynmptotically gauge theory.

The idea of dynamical symmetry breaking is very simple. Imagine an asymptotically free gauge theory, such as QCD. The QCD coupling constant is very small at high energies, *e.g.*, $\alpha_s(M_{\rm Planck}) \simeq 0.03$. The coupling grows as the energy goes down, and eventually becomes infinite at the QCD scale $\Lambda \sim 200$ MeV. The QCD scale is a *derived* energy scale from the initial coupling constant $\Lambda \sim M_{\rm Planck} e^{-2\pi/b_0\alpha_s(M_{\rm Planck})}$ where b_0 is the beta-function coefficient. Because of the expontential factor of the perturbative coupling at the Planck scale, the QCD scale is *much* smaller than the Planck scale. When the QCD becomes strong, the chiral symmetry of the up, down, and strange quarks is broken dynamically, and a mass is generated for the proton. This is why proton mass is so much smaller than the Planck scale; it truly *explains* the hierarchy. Put in more general terms, if a symmetry breaks due to an asymptotically free gauge theory becoming strong (Fig. 1), it is called dynamical symmetry breaking, and the scale of the symmetry breaking is naturally *much* smaller than the fundamental energy scale.

Can we use this idea to explain the smallness of the supersymmetry breaking scale? Dynamical breaking of supersymmetry, dynamicaly supersymmetry breaking, was proposed as a promising way to explain the hierarchy most clearly by Witten.[2] To use this idea in realistic model building, however, we needed to understand non-perturbative dynamics of supersymmetric gauge theories. Even though important progress had been made in mid-80's, the explosive progress had to wait until a series of works by Seiberg in mid-90's (see an excellent review by Intriligator and Seiberg[3]). Now we have many uncontroversial models of dynamical supersymmetry breaking. Probably the simplest model of dynamical supersymmetry breaking is a supersymmetric $SO(10)$ gauge theory with only one matter multiplet in the spinor representation **16**.[4]

Now that we know that supersymmetry can be dymamically broken, the next question is how the particles in the supersymmetric standard model learn that supersymmetry is broken. This is the issue of so-called "mediation" mechanism. Once they acquire supersymmetry breaking effects, such as mass differences between the standard model particles and their superpartners at a few hundred GeV scale, m_H^2 can acquire a negative value and the electroweak symmetry can get broken. On the other hand, phenomenology depends on the details of the mass spectrum, and hence on the details of the mediation mechanism. In the past few years, there has been a tremendous progress in discovering new mediation mechanisms.

The simplest idea for mediation is to *do nothing*. Even if there is apparently no interaction between the supersymmetric standard model and the sector which breaks supersymmetry dynamicaly (and is hence called "hiden sector"), there is at least gravity. If the dynamical supersymmetry breaking occurs at an energy scale Λ_{SUSY}, the gravitational effects induce su-

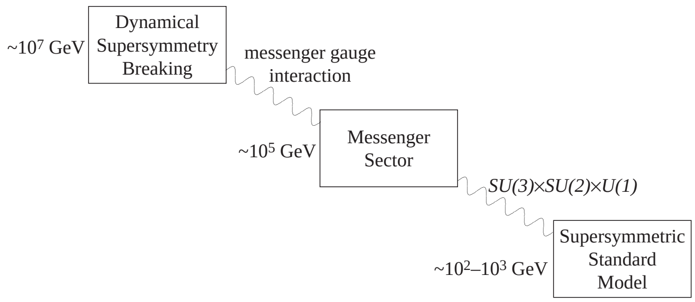

Figure 2: The original model of gauge mediation.[7]

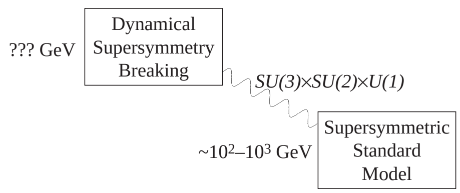

Figure 3: Direct gauge mediation models.[8]

persymmetry breaking masses of scalar quarks and leptons at order $\Lambda_{SUSY}^2/M_{\rm Planck}$. This option, however, had been regarded problematic, because the gravitational effects were believed to generate masses for gauginos (superpartners of gauge bosons) less than meV.[5] Recently, it was discovered that there is a quantum contribution to the gaugino masses due to the superconformal anomaly ("anomaly mediation")[6]

$$m_\lambda = \frac{\alpha}{2\pi} b_0 \frac{\Lambda_{SUSY}^2}{M_{\rm Planck}}. \tag{9}$$

This is an interesting formula because it gives an unconventional mass spectrum among gauginos $M_2 < M_1 < M_3$, which leads to new phenomenology.

Another approach is to use the standard model gauge interactions themselves to mediate the supersymmetry breaking effects ("gauge mediation").[7] In this scheme, the sector which breaks supersymmetry dynamically at $\Lambda_{SUSY} \sim 10^7$ GeV is coupled to another sector called "messenger sector" by a new gauge interaction, which causes particles in the messenger sector to acquire masses, both supersymmetric and supersymmetry breaking ones at around 10^5 GeV. The messenger particles carry the standard model gauge quantum numbers and their loops induce gaugino masses and scalar quark, lepton masses in the supersymmetric standard model at 10^2–10^3 GeV. This is a beautiful mechanism that generates the scalar masses according to their gauge quantum numbers; it makes, for instance, $\tilde{d}$, $\tilde{s}$, $\tilde{b}$ degenerate and avoids the would-be flavor-changing effects by the superGIM mechanism. But aesthetically, having three separate somewhat decoupled sectors was unpleasant. It turned out that the sector which breaks supersymmetry dynamically and the supersymmetric standard model can be coupled directly by the standard model gauge interactions ("direct gauge medation").[8] Since these models predict definite superparticle spectra, they can be tested once superparticles are found.[a]

What about fermion mass hierarchy? As discussed with the electron mass, smallness of the fermion masses is protected against self-energy corrections thanks to chiral symmetries. This point can be used in model building. The idea is to assume that some of the chiral symmetries (flavor symmetry) are indeed good, and hence first- and second-generation masses (possibly including τ, b) are forbidden in the symmetry limit, while the top quark mass is allowed. Once

[a] After this talk, there appeared many more proposals on supersymmetry breaking. See my brief review[9] for other ideas.

Table 1: A simple $U(1)$ flavor charge assignment which explains the fermion masses and mixings.[11] The top row shows the $SU(5)$ grand-unified multiplets.

	10	**5**	**1**
generation	Q u^c e^c	L d^c	N^c
1st	+2	0	0
2nd	+1	0	0
3rd	0	0	0

the flavor symmetry is broken by a small amount, the lighter masses are generated. A simple example is to have an approximate $U(1)$ symmetry, and assign the charges as given in Table 1 to the standard model particles.[10] We assume that the flavor symmetry is broken by a small parameter $\epsilon(-1) \sim 0.04$ which carries the $U(1)$ charge -1. Then one can write down the Yukawa matrices in power series expansion in ϵ so as to conserve the $U(1)$ charge,

$$(Q_1, Q_2, Q_3) \begin{pmatrix} \epsilon^4 & \epsilon^3 & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & \epsilon \\ \epsilon^2 & \epsilon & 1 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} \tag{10}$$

for the up quarks, and similarly for the others. Note that the symmetry requirements do not fix the precise coefficients in the power series in ϵ, and there are unknown $O(1)$ constants in every matrix elements which we cannot predict. But given that uncertainty, this pattern reproduces the observed quark, lepton masses and mixings including the solar and atmospheric neutrino oscillation data. The idea can also be extended to non-abelian flavor symmetries which have virtues of suppressing supersymmetric flavor-changing effects by keeping (at least) first- and second-generation scalars degenerate (see, *e.g.*, my review on models[12] and references therein).

The approximate flavor symmetry hence allows a successful model building, but it does not explain the origin of the small flavor symmetry breaking parameter ($\epsilon \sim 0.04 \ll 1$ in the above example). There are various ideas to explain the origin of the small number in this context. One of them is to use radiative breaking $\epsilon \sim g^2/16\pi^2$. This one-loop factor arises in the context of anomalous $U(1)$ gauge symmetry in string theory[13] or ordinary loop factor in the gauge mediated models.[14] The other possibility is to generate the small number dynamically as in the case of supersymmetry breaking.[15] It may also be due to the compositeness of **10** multiplets.[16]

It is noteworthy that the supersymmetric models with approximate flavor symmetry allow interesting flavor-changing effects. For instance, among two generations of down quarks, the mass matrix is given by

$$(Q_1, Q_2) \begin{pmatrix} m_d & m_s\lambda \\ m_s\lambda & m_s \end{pmatrix} \begin{pmatrix} d_1 \\ d_2 \end{pmatrix}, \tag{11}$$

where the hierarchy $m_d \ll m_s\lambda \ll m_s$ with $\lambda \sim 0.22$ may be a consequence of a $U(1)$ flavor symmetry. The corresponding mass matrix for the squarks then would be

$$m_{SUSY}(\tilde{Q}_1, \tilde{Q}_2) \begin{pmatrix} am_d & bm_s\lambda \\ cm_s\lambda & dm_s \end{pmatrix} \begin{pmatrix} \tilde{d}_1 \\ \tilde{d}_2 \end{pmatrix}, \tag{12}$$

where the flavor symmetry does not fix the $O(1)$ coefficients a, b, c, d. Going to the basis where the quark mass matrix is diagonalized by the Cabibbo rotation, the squark mass matrix is also rotated by the same amount and we find the new mass matrix in this basis

$$m_{SUSY}(\tilde{Q}_1, \tilde{Q}_2) \begin{pmatrix} (a-b-c+d)m_d & (b-d)m_s\lambda \\ (c-d)m_s\lambda & dm_s \end{pmatrix} \begin{pmatrix} \tilde{d}_1 \\ \tilde{d}_2 \end{pmatrix}. \tag{13}$$

Unless there is any particular reason for $b = c = d$, there remains off-diagonal elements in the squark mass matrix, which feeds into flavor-changing loop diagrams (see Gabbiani *et al*[17] for an extensive analysis of such flavor-changing effects). For instance, it may contribute to ϵ'/ϵ at an interesting level,[18]

$$\frac{\epsilon'}{\epsilon} \sim 3 \times 10^{-3} \left(\frac{500 \text{ GeV}}{m_{SUSY}}\right)^2 \Im(b-a). \tag{14}$$

Similar contributions appear in neutron and electron electric dipole moments and $\mu \to e\gamma$.

One interesting question is how exactly does a symmetry break dynamically in gauge theories. For instance in QCD, it is known that the $SU(3)_L \times SU(3)_R$ flavor symmetry is broken dynamically to $SU(3)_V$, and the corresponding Nambu–Goldstone bosons are the octet of pseudo-scalar mesons $\pi^\pm$, π^0, $K^\pm$, K^0, $\overline{K}^0$, η. Before discussing microscopic mechanism for the flavor symmetry breaking, let me remind you of a well-known qualitative argument for confinement in gauge theories due to monopole condensation by 't Hooft and Mandelstam.[19] In Type-II superconductors, there is a condensate of Cooper pairs (electric condensate), which causes magnetic fields to be squeezed in flux tubes. If you could place a pair of magnetic monopoles inside a superconductor, the magnetic field flux should be in a flux tube stretched between the monopoles and hence the energy increases linearly with the distance. This is nothing but the confinement. If you interchange electric and magnetic everywhere, you find that, in a system with a magnetic monopole condensate, the electric fields between charged particles are squeezed in flux tubes and the potential energy increases linearly with distance, and hence the electric charges are confined. This mechanism was shown to be operative at least in $N = 2$ supersymmetric theories in a beautiful analysis by Seiberg and Witten.[20] When there are quarks coupled to these theories, it turns out that the magnetic monopoles acquire flavor quantum numbers. For instance in $Sp(n_c)$ gauge theories with n_f quarks, the magnetic monopoles belong to the spinor representation of the $SO(2n_f)$ flavor symmetry. When they condense, not only they cause confinement, but also break the flavor symmetry from $SO(2n_f)$ to $U(n_f)$.[21]

3 Conclusions

I have hopefully convinced you that the dynamical symmetry breaking is a very natural concept in gauge theories and is useful in explaining hierarchies. Since the dynamics of supersymmetric gauge theories is understood quite well by now, we can use them to construct realistic, asthetically appealing models. When applied to mechanims of supersymmetry breaking and flavor symmetry breaking, different models predict different superparticle spectra and flavor-changing effects, and hence can be tested experimentally.

Acknowledgments

I thank the patience of organizers waiting for my writeup. This work was supported in part by the Department of Energy under contract DE–AC03–76SF00098, and in part by the National Science Foundation under grant PHY-95-14797.

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